

# Design Guide memory<sup>®</sup>-steel re-bar 10 & 16

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## Index of symbols

### Latin letters

$A_{SMA}$	Cross-sectional area of re-bar
$A_s$	Total cross-sectional area of reinforcement
$a_s$	Reinforcement area per metre
$A_p$	Total cross-sectional area of prestressing steel
$b$	Width of concrete cross-section
$d$	Effective depth of reinforcement
$d_f$	Effective depth of re-bar
$E_c$	Elastic modulus of concrete
$E_{SMA}$	Elastic modulus of memory <sup>®</sup> -steel
$F_c$	Concrete compressive force
$f_{cd}$	Design concrete compressive strength
$f_{sd,SMA}$	Design tensile strength of re-bar
$F_{sd,SMA}$	Tensile force of memory <sup>®</sup> -steel for cross-sectional analysis
$F_{p,0}$	Prestressing force of memory <sup>®</sup> -steel directly after activation at $t = 0$
$F_{p,\infty}$	Prestressing force of memory <sup>®</sup> -steel after relaxation at $t = \infty$
$f_{pk}$	Characteristic value of the tensile strength of prestressing steel
$f_{p0.1k,sd}$	Characteristic value of the 0.1% yield stress of prestressing steel
$F_s$	Tensile force in reinforcement cross-section
$h_c$	Thickness of concrete slab
$I$	Moment of inertia
$l$	Concrete slab/beam span
$l_b$	Anchorage length
$M_{Ed}$	Design bending moment
$M_{p,BZ}$	Prestressing moment from memory <sup>®</sup> -steel in construction state
$M_{p,GZ}$	Prestressing moment after relaxation (für for limit state calculation at $t = 0$ )
$M_{Rd}$	Design value bending resistance
$m_{Rd}$	Design value bending resistance of concrete slab, per metre
$P_0$	Prestressing force of a tendon at $t = 0$
$P_\infty$	Prestressing force of a tendon at $t = \infty$
$V_{Ed}$	Design value shear force
$V_{Rd}$	Design value shear resistance
$w_{eff}$	Existing deflection
$w_{all}$	Allowable deflection
$x$	Depth of bending compression zone
$z$	Lever arm

Note: Shape memory alloy (abbreviation SMA)

### Greek letters

$\gamma_p$	Safety value for prestressing steel
$\varepsilon_0$	Pre-strain of a tendon

$\varepsilon_c$	Concrete compressive strain
$\varepsilon_{p,0}$	Pre-strain of re-bar in factory
$\varepsilon_{p,sd}$	Design value of elongation at break of reinforcing steel or prestressing steel
$\varepsilon_s$	Reinforcement steel strain
$\varepsilon_{SMA}$	Strain of re-bar
$\varepsilon_{SMA,u}$	Elongation at break of re-bar
$\varepsilon_{ud}$	Design value of elongation at break of reinforcing steel or prestressing steel
$\Delta\varepsilon_{SMA}$	Increase of memory <sup>®</sup> -steel strain due to length change
$\Delta\sigma_{SMA}$	Increase of memory <sup>®</sup> -steel stress
$\sigma_{p,0}$	Initial memory <sup>®</sup> -steel prestressing directly after activation
$\sigma_{p,\infty}$	Long-term memory <sup>®</sup> -steel prestressing after relaxation

# 1 General information

## 1.1 Introduction

If all relevant product specifications of the manufacturer (memory<sup>®</sup>-steel, mortar and additional products) and standard specifications (roughness, concrete pre- and post-treatment etc.) are complied with, the conventional codes of structural design for reinforced and prestressed concrete structures can be applied.

The ribbed steel **memory<sup>®</sup>-steel re-bar 10 & 16** is placed in the Sika concrete replacement mortar or cast in a concrete groove. Only the tested Sika system mortars (R3 and R4 mortars for structural repairs) are authorised for use. A rigid bond is assumed for the design.

Design proposals for bending and shear reinforcements are explained below, both in the construction stage, in serviceability, and ultimate limit states. Design examples are shown for better understanding.

## 1.2 Basis of calculation

The design is carried out according to the standard codes for reinforced and prestressed concrete. For certain applications, masonry standards can be used as a basis.

Material parameters can be found in the currently valid and country-specific data sheets for the respective products (available at [www.re-fer.eu](http://www.re-fer.eu)). Values used in the design examples may deviate from the valid material parameters due to material optimisations and must be checked accordingly. The engineering department at re-fer AG will be happy to assist you with any uncertainties or special design situations. For further information, please visit our website (references, technical data sheets, processing guidelines, tender texts, test reports) or contact our technical service department directly.

The locally applicable standards and regulations must always be observed.

## 1.3 memory<sup>®</sup>-steel

### 1.3.1 Load-bearing behaviour

In the case of bending reinforcement, the prestressing forces in the re-bar system are introduced into the existing construction via the end anchorage zone as a result of the bond between the mortar and concrete and, if necessary, the end hooks. In most cases, the memory<sup>®</sup>-steel is activated and thus prestressed after the mortar has hardened in the anchoring area at both ends by means of a gas flame at a temperature of 300°C.

After heating and subsequent cooling, re-bar is also embedded in the mortar in the free length between the anchorage zones and can thus act as a composite with the existing structure. Due to the bond between re-bar and the mortar as well as the existing concrete structure, the standard design codes for reinforced and prestressed concrete can be applied. Due to the gradual application, the existing concrete is prestressed and the mortar subsequently applied in the free length is not.

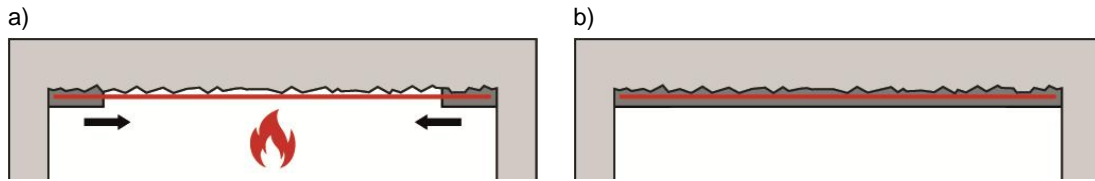


Figure 1: a) Prestressing forces introduced into anchorage zones, b) subsequent complete mortaring

Alternatively, the bars can also be completely grouted in advance and subsequently activated using electrical resistance heating. In this case, the prestressing is constant over the entire bar length. re-bar must be electrically separated from the internal reinforcement. Small concrete openings or bent-up end hooks on re-bars, for example, serve as connection points. These are filled with mortar or cut off at the end and sealed if necessary.

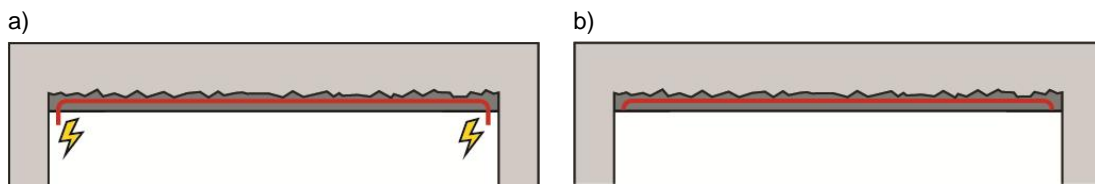


Figure 2: a) Electrical heating, prestressing over the entire length, b) cut-off end hooks

The same principles also apply to other applications of re-bar, such as use as shear reinforcement or for confinement.

### 1.3.2 Material behaviour

memory®-steel has no pronounced linear elastic, ideally plastic material behaviour. A small proportion of elastic deformation with a modulus of elasticity of  $160 \text{ kN/mm}^2$  ( $E_{SMA}$ ) quickly transforms into a 'pseudo-plastic' deformation. The crystal lattice of the material changes from an austenitic to a martensitic structure. This transformation of the crystal structure can be reversed to a certain extent at a later stage by means of heating.

re-bar is pre-strained in the factory to a remaining elongation  $\varepsilon_{p,0}$  and delivered to the construction site. If re-bar is anchored to the substrate (preventing back deformation) and heated, a prestress ( $\sigma_{p,0}$ ) is introduced into the substrate via the anchoring (A). If re-bar has no anchoring, the bar contracts again and cancels out parts of the initial pre-strain (B).

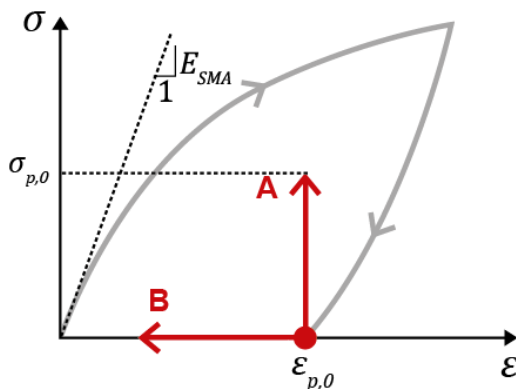


Figure 3: Schematic stress-strain diagram for pre-straining of re-bar

The material behaviour after prestressing is of interest for static structural or cross-section analyses:

- The prestressing  $\sigma_{p,0}$  is reduced by 15% over 50 years due to relaxation ( $\sigma_{p,\infty}$ ).
- Further loading or deformation of the structure results in an increase in stress  $\Delta\sigma_{SMA}$  via the bond between re-bar and the mortar/concrete.
- Up to a stress increase of 50 N/mm<sup>2</sup>,  $E_{SMA}$  is again 160 kN/mm<sup>2</sup>, but then drops to a value of around 70 kN/mm<sup>2</sup>
- The maximum tensile strength is >700 N/mm<sup>2</sup>, which is achieved at elongations at break of over 20%. At these stresses and strains, however, the material becomes very soft (very high ductility). Such high strains are hardly achieved for structural design.
- A maximum tensile strength  $f_{sd,SMA}$  of 520 N/mm<sup>2</sup> is therefore used for the design and no further increase in stress is assumed up to the design value of the elongation at break  $\varepsilon_{SMA,u}$  of 10%.

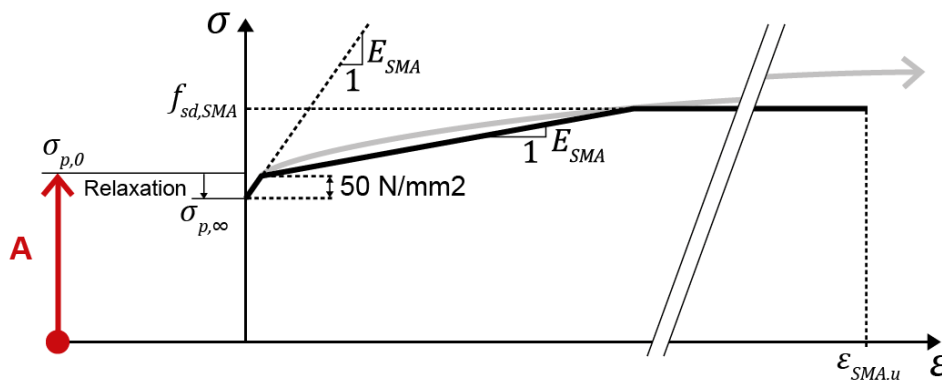


Figure 4: Stress-strain diagram of re-bar, after prestressing on the structure

The complex inclusion of the initial modulus of elasticity of 160 kN/mm<sup>2</sup> in a stress increment of 50 N/mm<sup>2</sup> makes sense for detailed analyses of structures subjected to dynamic fatigue loading. In this case, the increase in stress should not exceed 50 N/mm<sup>2</sup> under constant permanent loading.

For the simple design of the ultimate limit state, a constant  $E_{SMA}$  of 70 kN/mm<sup>2</sup> can be used for the sake of simplicity. This slightly underestimates the effective strength and stiffness of the material and represents a conservative simplification.

## 1.4 Theoretical design principles

As standard, the reinforcements in the end areas are anchored to the load-bearing substrate and the centre area is heated. The resulting prestressing force is transferred to the supporting structure via the end anchoring. For future loads, the reinforcement is therefore already actively load-bearing and at a defined stress level, which has various advantages - especially under permanent service loads. In order to utilise the resulting prestressing moment effectively, for example in terms of deflection, the force application points must be selected correctly. In the case of a simple bending beam, the deformation capacity of the structure is given, and the prestressing force acts directly against deflections. In the case of continuous beams or restraint floor slabs, the effect is reduced, as the deformability of the structure is reduced, and prestressing forces are partially transferred directly to



other components. It should also be ensured that the prestressing moments do not increase existing moments or support forces if they are incorrectly arranged. The general rules and procedures for the design of prestressed concrete must be observed here.

## 2 Dimensioning re-bar 10 & 16

### 2.1 State of construction

At construction state, it is important to check for possible cracking on top of the slab due to prestressing. The initial memory®-steel prestressing  $\sigma_{p,0}$  is applied in this case. The prestressing can be set as a constant bending moment  $M_{p,BZ}$  between the anchorages, to be compared with the cracking moment.

$$M_{p,BZ} = F_{p,0} * z = \sigma_{p,0} * A_{SMA} * z \quad (1)$$

( $A_r$  = area re-bar,  $z$  = lever arm)

### 2.2 Serviceability limit state SLS

For the serviceability limit state over a long period, the initial prestressing  $\sigma_{p,0}$  must be reduced due to relaxation. Over a period of 50 years this can be estimated at 15%. The following equation applies:

$$\sigma_{p,\infty} \approx \sigma_{p,0} * 0.85 \quad (2)$$

The constant bending moment  $M_{p,GZ}$  between the anchorages can therefore be described as:

$$M_{p,GZ} = F_{p,\infty} * z = \sigma_{p,\infty} * A_{SMA} * z \quad (3)$$

Deformation reductions or limitations can be calculated using this prestressing moment.

For the service state of dynamically loaded structures stress increases must be limited to 50 N/mm<sup>2</sup>:

$$\Delta\sigma_{SMA} \leq 50 \text{ MPa} \quad (4)$$

### 2.3 Ultimate limit state ULS

In the ultimate limit state, re-bar undergoes an additional strain  $\Delta\epsilon_{SMA}$  and stress  $\Delta\sigma_{SMA}$ , which is added to the initial prestressing. The change in stress is limited up to the maximum design tensile strength of the material.

Thanks to the stress-strain compatibility of re-bar with the load-bearing concrete base, a force equilibrium can be calculated. The factors shown are described in the list of letters.



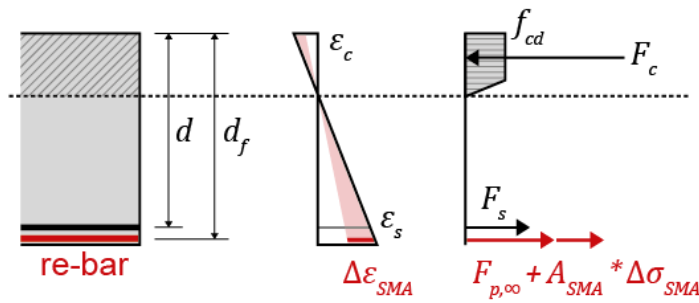


Figure 5: Schematic Representation for cross-sectional analysis of the ultimate limit state

The force equilibrium is then carried out with an equivalent force in the re-bar, which is made up as follows. For simplicity, a reduced elastic modulus  $E_{SMA}$  of 70 kN/mm<sup>2</sup> is applied. The final force must be smaller as the maximum tensile force  $f_{sd,SMA}$  of re-bar.

$$F_{sd,SMA} = F_{p,\infty} + A_{SMA} * (\sigma_{p,\infty} + \Delta\epsilon_{SMA} * E_{SMA}) \leq A_{SMA} * f_{sd,SMA} \quad (5)$$

## 2.4 Anchoring of re-bar

The method statements (processing guidelines) and data sheets of re-fer provide guide values for the anchorage lengths of re-bar. The anchoring zones depend on the expected tensile forces, bar diameters and application type. In addition, standard specifications regarding adhesive tensile strength, roughness and preparation of the substrate etc. must be observed. Only approved R3 and R4 mortars are to be used for concrete repairs. An adhesive tensile strength of 1.5 N/mm<sup>2</sup> is usually assumed for the concrete substrate.

Bending reinforcements must always be anchored behind the zero-moment line and, depending on the situation, it must be checked whether the tensile reinforcement is sufficiently anchored over the supports. The introduction of the prestressing force into the concrete base is normally achieved via the pure mortar bond. For this purpose, the adhesive tensile resistance of the concrete base can also be used to calculate the required anchoring area for the force transfer. Alternatively, end hooks on re-bar can be used to transfer the forces selectively into the core concrete. Special anchoring using bolted or welded steel profiles is also possible.

## 2.5 Cross-sectional analysis software

To use memory®-steel in cross-section analysis tools, certain presetting and entries for the material parameters are required. The programmes usually calculate with bilinear or even trilinear working diagrams of the prestressing steel (see schematic stress-strain relationship):

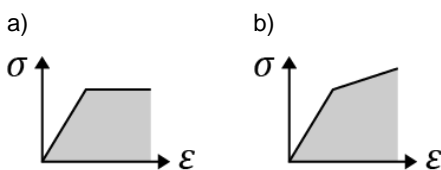


Figure 6: a) and b) Examples of bilinear behaviour of calculation models

To start with, you need to check in the relevant programme which principle the software uses to calculate in order to use the appropriate material parameters later on. Certain default settings can also be made.

- **Safety factors:** For prestressing steel, the safety factor  $\gamma_p = 1.15$  is usually used in the ultimate limit state. The design value of the prestressing steel is therefore additionally reduced. At our own judgement, it is also possible for memory®-steel to set this coefficient to 1.00, as  $f_{sd,SMA}$  is already greatly reduced due to the application/mechanical principles and can be considered the design value.
- **Initial value of the prestressing force:** In some cases, a choice must be made as to whether to design with a full initial prestress or whether to include long-term losses. (see considerations in Chap. **Fehler! Verweisquelle konnte nicht gefunden werden.**, 2.2 and 2.3)

Afterwards, a new material is now created. Information on the material type and mechanical properties must be defined. The following table is an example of bilinear material behaviour (parameter designations may differ depending on the software). It should always be ensured that the design software performs a calculation «with bond». Otherwise, no stress increase to the initial prestress is calculated in the re-bar and the contribution to the load-bearing safety is set very conservatively.

Parameter	re-bar 16	re-bar 10
$f_{pk}$ or $f_{p0.1k,sd}$ etc.	520 N/mm <sup>2</sup>	520 N/mm <sup>2</sup>
$\varepsilon_{p,sd}$ or $\varepsilon_{ud}$	100‰	100‰
$E_s$	70 kN/mm <sup>2</sup>	70 kN/mm <sup>2</sup>
$A_p$	211.2 mm <sup>2</sup>	89.9 mm <sup>2</sup>
Pre-straining $\varepsilon_0$	4.57‰ (heating with 300°C) 3.71‰ (heating with 200°C)	5.71‰ (heating with 300°C) 5.00‰ (heating with 300°C)
Loss factor $P \sigma_{p,\infty}/P_0$	0.85	0.85
Calculation with bond	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>

All values must always be compared with current data sheets.

**Pre-straining:** Normally, the design software determines the applied prestress based on the modulus of elasticity and the applied strain, which in conventional prestressed concrete is calculated on the basis of the elongation path of the prestressing strands and their total length. For re-bar, the prestressing strain must therefore be defined using the elastic material law, which results in the desired prestress  $\sigma_{p,0}$  for the entered modulus of elasticity:

$$\varepsilon_0 = \sigma_{p,0} / E_{SMA} \quad (6)$$

For more detailed calculations (e.g. for serviceability limit state with limited stress increase) according to trilinear behaviour, the basic principles from chapter 1.3.2 and 2.2 should be consulted. If the planner has any questions, it is recommended to contact the technical support of re-fer AG.

## 3 Planning notes

### 3.1 Corrosion protection

A known risk of prestressing steels is stress corrosion cracking (SCC) in the presence of chlorides, other salts and acids. re-bar is placed in a cementitious matrix, which serves as an alkali deposit for the internal reinforcement and as a protective layer against penetrating chloride ions. SikaTop® Armatec®-110 EpoCem® can be applied to re-bar and the internal reinforcement as a bonding agent and slight corrosion protection.

The standards for concrete construction (e.g. Switzerland SIA 262, Table 18) can be consulted for specifications regarding the concrete cover thickness depending on the exposure class of the component.

### 3.2 Fire protection

memory®-steel exhibits similar fire behaviour to conventional steel and significantly loses strength at around 400°C and reduces its prestressing to zero at approx. 350°C. If the minimum component dimensions and concrete cover are complied with in accordance with the local standard (e.g. Switzerland SIA 262, Table 16), protection in the event of fire is generally given. re-fer recommends increasing these values by a factor of 1.25 to 1.50, similar to what is often used for prestressed concrete components.

If fire protection cannot be guaranteed by the reinforcement cover, products such as the fire protection spray plaster SikaCem® Pyrocoat can also be applied to the finished surface to further increase the fire resistance.

### 3.3 Constructive rules

Local standards must also be consulted with regard to structural designs, reinforcement guides etc. For memory®-steel, the outside diameters of the screw couplers in particular must be taken into account. When planning re-bar U-profiles or end hooks, the minimum bending radius according to the data sheet and method statement apply.

**Caution:** No third-party products (construction foam, XPS, polystyrene and other chemical and chloride-based components) may be used; these can form aggressive decomposition products when re-bar is heated, which are hazardous to the respiratory tract and can lead to corrosion or acid attack on the steel. In general, the relevant method statement and specifications for mortar work, post-treatment or substrate preparation must be observed.

### 3.4 Other verifications

For other verifications, design- or execution-related questions, please refer to the local standard for reinforced concrete and prestressing steel, or masonry. The technical support of re-fer AG can also provide assistance.

## 4 Design examples

The dimensioning examples must always be compared with currently valid local standards. Deviations are reserved.

### 4.1 Shear and flexural strengthening of T-beam

Due to a change of use and additional loads, various T-beams in a factory building need structural strengthening. This calculation example illustrates the method for excessive deflection in the main span and strengthening for flexural and shear problems in an individual beam of this kind. Additional verifications are omitted in this example. The beams covered two spans of 12.00 and 8.00 m and were simply supported.



Figure 7: Two span beam in a factory building

The previous static forces (bending moments and shear forces) are shown below; there are no additional normal or torsion forces.

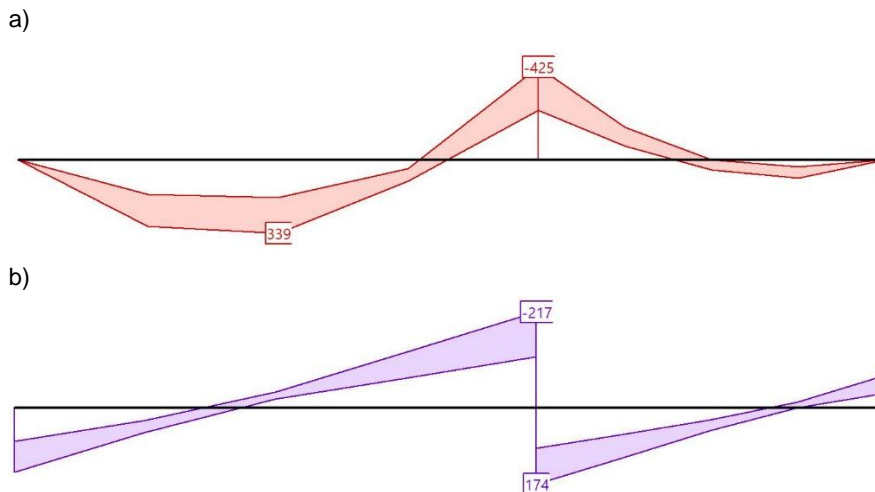


Figure 8: Internal forces at structural ultimate limit state a) Bending moment  $M_y$  [kNm] b) Shear loads  $V_z$  [kN]

In line with the original loadings, the beams were designed and reinforced as shown in Figure 9. The resultant deflection in the cracked concrete cross-section met the required standard specifications ( $w_{eff} = 32$  mm /  $w_{all} = 34$  mm).

Due to the client's new requirements, live loads are increased. A higher dead load also must be supported due to the additional mortar layer to be added. The resultant static forces for the structural safety ultimate limit state are as follows:

	Previous internal forces	Previous resistances	New internal forces
<b>Bending moment [kNm]</b>	$M_{Ed}$ +339 -425	$M_{Rd}$ +355 -440	$M_{Ed}$ +449 -550
<b>Shear force [kN]</b>	$V_{Ed}$ 217	$V_{Rd}$ 230	$V_{Ed}$ 285

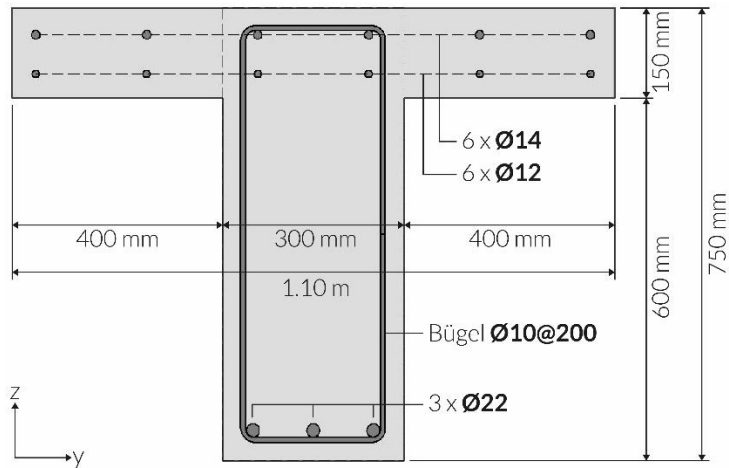
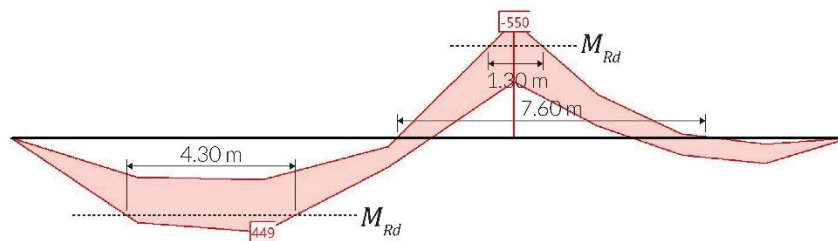


Figure 9: Existing cross-section of T-beams

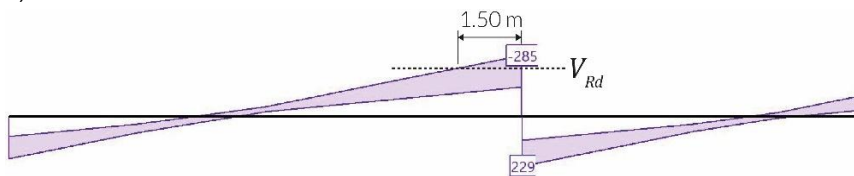
#### 4.1.1 Verification of structural safety at ultimate limit state:

Firstly, the structural safety ultimate limit state is investigated. The new internal forces are also shown in detail below.

a)



b)


Figure 10: New internal forces at ultimate limit state a) Bending moment  $M_y$  [kNm] b) Shear forces  $V_z$  [kN]

Due to the additional loads, a shear problem occurs in a region about 1.5 m wide adjacent to the central point of support. The missing transverse shear strength of ca. 55 kN/m' is accommodated using re-bar 10 U-profiles. For simplicity, only the prestressing force (no strain increase up to shear failure) on the double shear stirrups is assumed.

$$V_{Rd,s} = \frac{2 \cdot \sigma_{p,\infty} \cdot A_f}{s} \cdot z \cdot \cot(45^\circ) = \frac{2 \cdot 350 \frac{N}{mm^2} \cdot 0.85 \cdot 89.9 \text{ mm}^2}{0.5 \text{ m}} \cdot \sim 0.7 \text{ m} \cdot \cot(45^\circ) = 75 \text{ kN/m'}$$

Accordingly, a total of three re-bar 10 U-profiles at a 0.5 m interval are necessary to strengthen the region. The stirrups are guided around the existing, roughened concrete surface and over the additional longitudinal re-bar. They are then embedded in sprayed mortar / grouted in the flange (anchorage over the neutral axis). The re-bar shear stirrups are electrically heated/activated from above. Spacers are installed to ensure that there is no contact with the existing reinforcement (electric tension loss during heating process).

In the larger sub-span, the new bending effect exceeds the previous resistance by some 94 kNm. Over the whole span, three re-bar 16 are installed on the bottom side of the web and embedded in sprayed mortar. Across the central support, the negative bending moment exceeds the permitted load over a length of ca. 1.3 by approx. 110 kNm. In that zone, four re-bars 10 are laid in fresh concrete cover (Note: anchorage of strengthening behind the zero-moment line). The strengthening bars are grouted in the anchorage region and heated after hardening, e.g. with a gas burner. Finally, the remaining zones are also embedded.

Flexural verification of the new cross-section can be done with standard design software. The new resistance levels are listed in the table below.

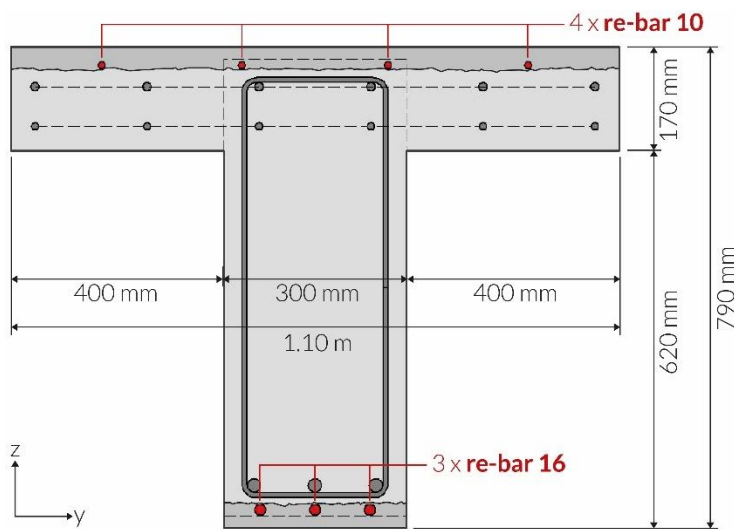


Figure 11: new cross-section of T-beams with re-bar flexural strengthening

	Previous internal forces		Previous resistance		New internal forces		New resistance	
Bending moment [kNm]	$M_{Ed}$	+339	$M_{Rd}$	+355	$M_{Ed}$	+449	$M_{Rd}$	+569
		-425		-440		-550		-553
Shear force [kN]	$V_{Ed}$	217	$V_{Rd}$	230	$V_{Ed}$	285	$V_{Rd}$	315

The following input parameters are used, amongst others, for the modelling:



#### Attributes of “new tendon”:

- Pre-straining  $\varepsilon_0 = 0.57\%$  for re-bar 10 and  $0.46\%$  for re-bar 16 (which gives theoretical prestressing of: *Elastic modulus* \*  $\varepsilon_0 = 400 \text{ N/mm}^2$ , and  $320 \text{ N/mm}^2$ )
- Prestressing with bond
- Loss factor  $P_\infty/P_0 = 0.85$  (relaxation)

#### Material properties:

- *Elastic modulus* =  $70 \text{ kN/mm}^2$  (re-bar elastic modulus after activation)
- $f_{p0.1k} = 520 \text{ N/mm}^2$  (Design value reduced by safety factor)
- $\varepsilon_{ud} = 30\%$

#### 4.1.2 Verification at service load level:

By installing prestressed strengthening elements embedded in mortar, crack openings are limited at the surface, and load is removed from the existing reinforcement. In addition to the improved durability, this example also investigates the deflection. Due to the new loads, the vertical deflection in the large span is calculated at about 39 mm. Flexural strengthening with three re-bar 16 implies a constant bending moment which counteracts the deflection. The resulting 5 mm ( $w_{eff} = 39 \text{ mm} / w_{all} = 34 \text{ mm}$ ) should be eliminated with this measure.

The deformation of the statically indeterminate system implied by the prestressing can be calculated in various ways. Here, the principle of virtual work for the statically indeterminate system is used. As a basic system (BS), an articulated joint is introduced at the central support. For simplicity, the prestressing in the negative bending region is not included, though it would also have a positive effect.

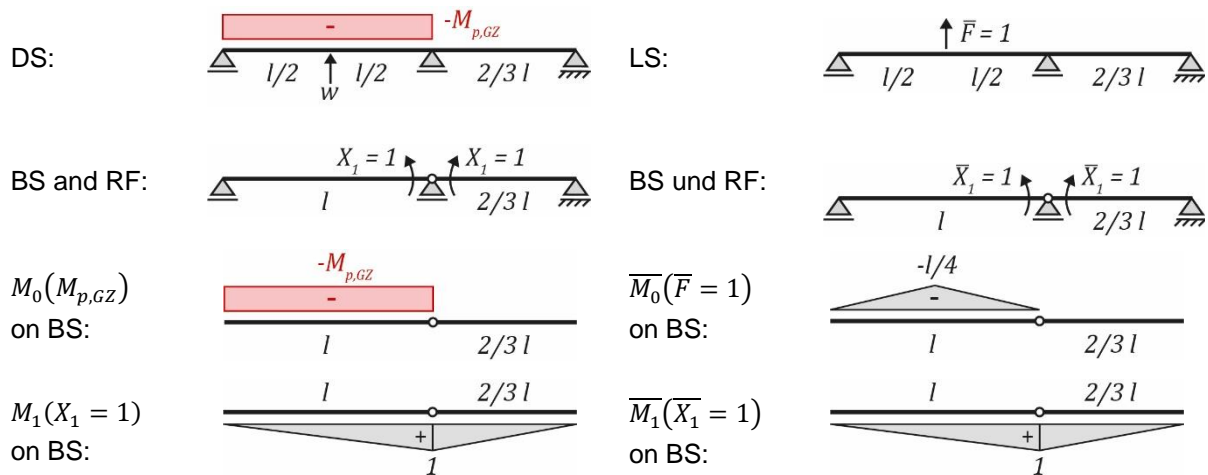


Figure 12: Simplification and reduction of the statically indeterminate system and principle of virtual work

$$\delta_{10} = \int M_1 * \frac{M_0}{E_c I} dx = \frac{1}{2} * (+1) * (-M_{p,GZ}) * \frac{l}{E_c I} + 0 = -\frac{M_{p,GZ} * l}{2 * E_c I}$$

$$\delta_{11} = \int M_1 * \frac{M_1}{E_c I} dx = \frac{1}{3} * (+1)^2 * \frac{\left(1 + \frac{2}{3}\right) l}{E_c I} = \frac{5 * l}{9 * E_c I}$$

$$\delta_{10} + X_1 * \delta_{11} = 0 \rightarrow X_1 = -\frac{\delta_{10}}{\delta_{11}} = \frac{9}{10} M_{p,GZ}$$

The deformation  $w$  can be deduced from this as follows:

$$\begin{aligned} w &= \int \overline{M}_0 * \frac{M_0}{E_c I} dx + X_1 * \int \overline{M}_0 * \frac{M_1}{E_c I} dx \\ &= \frac{1}{2} * \left(-\frac{l}{4}\right) * (-M_{p,GZ}) * \frac{l}{E_c I} + \left(\frac{9}{10} M_{p,GZ}\right) * \frac{1}{4} * \left(-\frac{l}{4}\right) * (+1) * \frac{l}{E_c I} \\ &= \frac{M_{p,GZ} * l^2}{E_c I} * \left(\frac{1}{8} - \frac{9}{160}\right) = \frac{11 * M_{p,GZ} * l^2}{160 * E_c I} \end{aligned}$$

Equation (3) gives the constant bending moment  $M_{p,GZ}$  across the 12.00:

$$M_{p,GZ} = F_{p,\infty} * z = \sigma_{p,\infty} * A_f * z = 3 * 320 \frac{N}{mm^2} * 0.85 * 211.2 mm^2 * \sim 0.66 m = 114 kNm$$

In addition, a reduced, cracked bending stiffness of the concrete cross-section is estimated ( $E_c I_{cracked} = E_c I / 3$ ) and included in the equation.

$$w = \frac{11 * M_{p,GZ} * l^2}{160 * \left(E_c I / 3\right)} = \frac{11 * 114 kNm * (12.00 m)^2}{160 * \frac{647'000 kNm^2}{3}} = 5.2 mm$$

The three re-bars installed to increase the structural safety consequently contribute to a reduction in the deflection of around 5 mm. The verification is achieved.

#### 4.1.3 Verification of anchorage zones:

The negative and positive bending resistances were determined by a cross-sectional analysis software. The maximum tensile force in the re-bar and a tensile adhesion strength of the concrete of 1.5 N/mm<sup>2</sup> is used to design the anchoring zone. The resistance is reduced by a safety factor of 1.5. Four re-bar 10 are applied for the negative bending. Out of this, the following calculation for the necessary bond length  $l_b$  results:

$$F_{p,i}(negative) = 4 * \sigma_{p,i} * A_f = 4 * 520 \frac{N}{mm^2} * 89.9 mm^2 = 187.0 kN$$

$$F_{p,i} \leq \frac{l_b * 1.10 m * 1.5 \frac{N}{mm^2}}{1.5} \rightarrow l_b = 170 mm$$

The strengthening measure is embedded entirely in mortar. The anchorage region is assumed to be 300 mm of length.

In the case of strengthening against positive bending, three re-bar 16 are mounted on the bottom side of the web (width 300 mm). Again, the total maximum tensile force of the re-bars is anchored.

$$F_{p,i}(positive) = 3 * \sigma_{p,i} * A_f = 3 * 520 \frac{N}{mm^2} * 211.2 mm^2 = 329.5 kN$$

$$F_{p,i} \leq \frac{l_b * 300 mm * 1.5 \frac{N}{mm^2}}{1.5} \rightarrow l_b = 1'098 mm$$

This value can be optimized by using special solutions. As an example, the effect of the vertical prestressing by three re-bar U-profiles is presented. The tensile adhesion strength (1.5 N/mm<sup>2</sup>)

increases due to the vertical force of the prestressed U-profile in double shear (relaxation prestressing force 0.85 / safety factor 1.5).

$$F_{p,i} = 329.5 \text{ kN} \leq \frac{l_b \cdot b \cdot \left( 1.5 \frac{N}{\text{mm}^2} + \frac{3 \cdot 2 \cdot \sigma_{p,\infty} \cdot A_f}{l_b \cdot b} \right)}{1.5} =$$

$$\frac{l_b \cdot 300 \text{ mm} \cdot \left( 1.5 \frac{N}{\text{mm}^2} + \frac{3 \cdot 2 \cdot 0.85 \cdot 350 \text{ N/mm}^2 \cdot 89.9 \text{ mm}^2}{l_b \cdot 300 \text{ mm}} \right)}{1.5} \rightarrow l_b = 742 \text{ mm}$$

Analogue to the intermediate support B, a shear strengthening with re-bar 10 U-profile is applied for support A, too. The anchorage zone is embedded in mortar over a length of 750 mm.

#### 4.1.4 Schematic drawing:

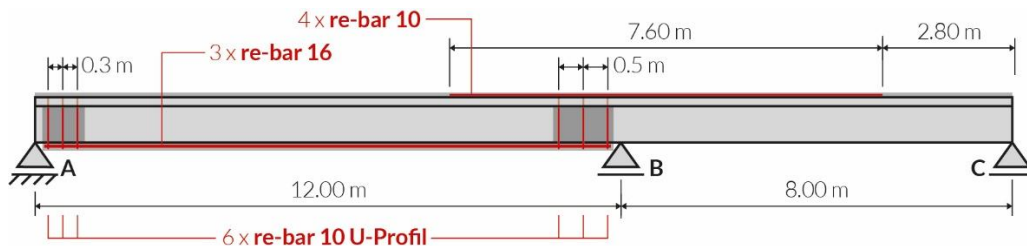


Figure 13: Sketch of strengthening works with re-bar longitudinal reinforcement and re-bar shear stirrups

The end regions of the re-bar flexural strengthening could also be made by conventional, slack-applied stirrups (steel B500B).

## 4.2 Further design examples

Further design examples follow.

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